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A NEW DESCRIPTION OF CONICS.

By PROF. J. W. NICHOLSON, Baton Rouge, La.

The locus of the first order, or right-line, is represented by the equation

$$y = ax + b.$$

This may have, with reference to the x -axis any one of three directions: it may be parallel to it, in which case $a = 0$; it may make an acute angle with it, in which case $a > 0$; it may make an obtuse angle with it, in which case $a < 0$. In the first case y is constant; in the second y is an increasing linear function of x ; in the third case y is a decreasing linear function of x . The equation may therefore be written under one of the following forms:

$$y = C, \quad y = Y, \quad y = \Lambda,$$

where C denotes a constant, Y an increasing linear function of x , Λ a decreasing linear function of x .

The locus of the second order, or conic section, is represented by the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

which by appropriate transformations is reducible to the form

$$y^2 = Px^2 + Qx + R,$$

and represents in general a parabola, an ellipse, or an hyperbola as

$$P = 0 < > 0.$$

Observing that the second member of the reduced equation can always be identified with the product

$$(a_1x + b_1)(a_2x + b_2) = a_1a_2x^2 + (a_1b_2 + a_2b_1)x + b_1b_2,$$

and that conversely it is in general resolvable into two real linear factors of the form $a_1x + b_1, a_2x + b_2$ we are justified in asserting the following propositions:—

I. The locus whose ordinate is a mean proportional between the corresponding ordinates of two given right lines is a conic.

II. Conversely, every conic can be generated by a point which moves so that its ordinate is a mean proportional between the corresponding ordinates of two suitably located right lines.

From these propositions the following corollaries flow:—

I. If a_1 or $a_2 = 0$, $P = 0$ and the conic is a parabola; conversely, every parabola can be represented by an equation of the form

$$y^2 = CY \text{ or } C\Lambda.$$

2. If α_1, α_2 are of opposite signs, $P < 0$ and the conic is an ellipse; conversely, every ellipse can be represented by an equation of the form

$$y^2 = YA.$$

3. If α_1, α_2 are of like signs, $P > 0$ and the conic is an hyperbola; conversely, every hyperbola can be represented by an equation of the form

$$y^2 = Y_1 Y_2.$$

These propositions furnish easy tests of the nature of any given locus of the second order. The following illustrative examples are added:—

a. To determine the figure of the section of a right circular cone by a plane.

VFG is the cone; *AHK* the plane; *FQR* a parallel circle intersecting the plane in *P*. Cut by a plane through *V* normal to the secant plane, intersecting the latter in *AJ*, the base in *FG*, and the parallel in *QR*, and join *PM*, *M* being the intersection of *AJ* and *QR*. Then *PM* is the ordinate of the required curve and

$$PM^2 = QM \cdot MR.$$

Now *MR*, the ordinate of *VG* relative to *AJ*, is increasing; hence the locus is an ellipse, a parabola, or an hyperbola, as *QM* is decreasing, constant, or increasing; that is, as the angle *GAJ* is greater than, equal to, or less than the vertical angle of the cone.

b. To find the locus of a point whose distance from a fixed circle measured on a radius equals its distance from a fixed diameter of that circle measured on a line of fixed direction.

Take the centre for origin and the fixed diameter for *y*-axis, and let the fixed direction make with the *x*-axis the angle α . Then the equation to the locus is

$$\begin{aligned} y^2 &= (r + x \sec \alpha)^2 - x^2 \\ &= [r + x(\sec \alpha + 1)] \cdot [r + x(\sec \alpha - 1)], \end{aligned}$$

and represents a parabola or an hyperbola as $\sec \alpha = > 1$.

[Professor Nicholson observes in a letter to the editors that the factors of $Px^2 + Qx + R$ will, if the axes are suitably chosen, be real when the conic is real, and imaginary when the conic is imaginary. The criteria of form in the latter case appear to fail.—ED.]